Suggested solutions to the Contract Theory resit exam on Feb. 24, 2012
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Question 1 (adverse selection)

This adverse selection model with two types is identical to one that we studied in the course.

A firm (the agent, A) interacts with a government procurement agency (the principal, P). A produces office material that P wants to purchase. A's cost of producing q units of office material is given by the function $C(q, \theta)$, where θ is an efficiency parameter. This function satisfies

$$C(0, \theta) = 0, \qquad C_q > 0, \qquad C_{qq} \geq 0, \qquad C_\theta > 0, \qquad C_{q\theta} > 0, \qquad C_{qq\theta} \geq 0.$$

The value for P of receiving q units of office material is given by the function $S(q)$, which satisfies

$$S'(q) > 0, \qquad S''(q) < 0, \qquad S(0) = 0.$$

The efficiency parameter θ can take two values: $\theta \in \{\underline{\theta}, \overline{\theta}\}$, with $0 < \underline{\theta} < \overline{\theta}$. A knows the value of θ perfectly. However, P only knows that

$$\Pr[\theta = \underline{\theta}] = \nu \qquad \text{and} \qquad \Pr\left[\theta = \overline{\theta}\right] = 1 - \nu,$$

with $0 < \nu < 1$. The procurement agency has all the bargaining power and makes a take-it-or-leave-it offer to the firm. A contract can specify the quantity q that A must produce and deliver and the payment t that A will receive. Suppose that P wants to offer different contracts to the two types of firms. P is risk neutral and its payoff, given a quantity q and a payment t, equals

$$V = S(q) - t.$$

A is also risk neutral and its payoff, given a quantity q and a payment t, equals

$$U = t - C(q, \theta).$$

A's outside option (the same for both types) would yield the payoff zero.

P offers a menu of two distinct contracts to A. As in the course, the contract variables are indicated either with "upper-bars" or with "lower-bars", depending on which type the contract is aimed at. P's problem is to choose $(\underline{t}, \underline{q}, \overline{t}, \overline{q})$ so as to maximize

$$\nu\left[S(\underline{q}) - \underline{t}\right] + (1 - \nu)\left[S(\overline{q}) - \overline{t}\right]$$

subject to the following four constraints:

$$\overline{t} - C\left(\overline{q}, \overline{\theta}\right) \geq 0, \tag{IR-bad}$$

$$\underline{t}-\mathbf{C}\left(\underline{q},\underline{\theta}\right)\geq\mathbf{0}, \tag{IR-good}$$

$$\overline{t}-\mathbf{C}\left(\overline{q},\overline{\theta}\right)\geq\underline{t}-\mathbf{C}\left(\underline{q},\overline{\theta}\right), \tag{IC-bad}$$

$$\underline{t}-\mathbf{C}\left(\underline{q},\underline{\theta}\right)\geq\overline{t}-\mathbf{C}\left(\overline{q},\underline{\theta}\right). \tag{IC-good}$$

a) Explain in words what each one of the four constraints says and why it must be satisfied at the optimum.

- The IR-bad constraint says that the less able type of agent (with $\theta=\overline{\theta}$) must, at least weakly, prefer the contract aimed at her to the outside option. Choosing the contract yields the utility $\overline{t}-C\left(\overline{q},\overline{\theta}\right)$ and the outside option yields the utility zero. If this condition was violated, the less able type of agent would not choose the contract that P wants her to choose, because the outside option yields a higher utility.
- The interpretation of the IR-good constraint is analogous to the one for IR-bad, but concerns the relatively able type (with $\theta=\underline{\theta}$).
- The IC-bad constraint says that the less able type of agent must, at least weakly, prefer the contract aimed at her to the contract aimed at the relatively able agent. This condition must be satisfied for the less able agent to choose the contract P wants her to choose. P must ensure that this condition is satisfied because P cannot observe the agent's type directly and therefore is unable to instruct the agent to pick one of the two contracts: each agent type must have an incentive to voluntarily choose the one aimed at her.
- The interpretation of the IC-good constraint is analogous to the one for IC-bad, but concerns the relatively able type.

b) Prove that incentive compatibility and Spence-Mirrlees ($C_{q\theta}>0$) imply monotonicity; that is, show that if the inequalities defining incentive compatibility hold and if the Spence-Mirrlees condition is satisfied, then the quantity offered to the $\underline{\theta}$-type agent is at least as large as the one offered to the $\overline{\theta}$-type agent.

- Incentive compatibility means that IC-bad and IC-good hold. Adding these two inequalities yields:

$$\left[\underline{t}-C\left(\underline{q},\underline{\theta}\right)\right]+\left[\overline{t}-C\left(\overline{q},\overline{\theta}\right)\right]\geq\left[\overline{t}-C\left(\overline{q},\underline{\theta}\right)\right]+\left[\underline{t}-C\left(\underline{q},\overline{\theta}\right)\right]$$

The t's cancel out, so the above inequality simplifies to

$$C\left(\underline{q},\overline{\theta}\right)-C\left(\overline{q},\overline{\theta}\right)\geq C\left(\underline{q},\underline{\theta}\right)-C\left(\overline{q},\underline{\theta}\right)$$

Rewriting again on integral form yields

$$\int_{\overline{q}}^{\underline{q}}C_q\left(q,\overline{\theta}\right)dq\geq\int_{\overline{q}}^{\underline{q}}C_q\left(q,\underline{\theta}\right)dq$$

Rewriting yet again, on double integral form, we obtain

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\overline{q}}^{\underline{q}} C_{q\theta}\left(q,\theta\right) dq d\theta \geq 0$$

By $\overline{\theta} > \underline{\theta}$ and the Spence-Mirrlees property $C_{q\theta} > 0$, the last inequality implies $\underline{q} \geq \overline{q}$, which is what we were asked to prove.

c) The first best optimal quantities are defined by $S'\left(\underline{q}^{FB}\right) = C_q\left(\underline{q}^{FB}, \underline{\theta}\right)$ and $S'\left(\overline{q}^{FB}\right) = C_q\left(\overline{q}^{FB}, \overline{\theta}\right)$, respectively. Assume that the constraints (IR-good) and (IC-bad) are lax at the second-best optimum (so that they can be disregarded). Show that, at the second-best optimum, the good type's quantity is not distorted relative to the first best ($\underline{q}^{SB} = \underline{q}^{FB}$) and that the bad type's quantity is distorted downwards ($\overline{q}^{SB} < \overline{q}^{FB}$).

- We are allowed to assume that (IR-good) and (IC-bad) are lax at the optimum. Given that, the problem can be written as: Choose $\left(\underline{t}, \underline{q}, \overline{t}, \overline{q}\right)$ so as to maximize

$$\nu\left[S\left(\underline{q}\right) - \underline{t}\right] + (1-\nu)\left[S\left(\overline{q}\right) - \overline{t}\right]$$

subject to the following two constraints:

$$\overline{t} - C\left(\overline{q}, \overline{\theta}\right) \geq 0, \qquad \text{(IR-bad)}$$

$$\underline{t} - C\left(\underline{q}, \underline{\theta}\right) \geq \overline{t} - C\left(\overline{q}, \underline{\theta}\right). \qquad \text{(IC-good)}$$

- *Claim*: At the optimum of the problem above, both constraints must bind. *Proof of claim*:
  - Suppose, per contra, that we have an optimum and that IR-bad is lax. Then we can lower $\overline{t}$, while still satisfying both constraints (IC-good will actually be relaxed), thereby increasing the value of the objective function (for this is decreasing in $\overline{t}$). But that is impossible, since we started at an optimum. Hence IR-bad must bind at an optimum.
  - Suppose, per contra, that we have an optimum and that IC-good is lax. Then we can lower $\underline{t}$, while still satisfying both constraints (IR-bad will not be affected), thereby increasing the value of the objective function (for this is decreasing in $\underline{t}$). But that is impossible, since we started at an optimum. Hence IC-good must bind at an optimum.

- Given that both constraints bind, we can replace the inequalities with equalities and then solve for $\underline{t}$ and $\overline{t}$. Doing this we get:

$$\overline{t} - C\left(\overline{q}, \overline{\theta}\right) = 0 \Rightarrow \overline{t} = C\left(\overline{q}, \overline{\theta}\right)$$

and

$$\underline{t} - C\left(\underline{q}, \underline{\theta}\right) = \overline{t} - C\left(\overline{q}, \underline{\theta}\right) \Rightarrow$$

$$\begin{aligned}\underline{t} &= C\left(\underline{q}, \underline{\theta}\right) + \overline{t} - C\left(\overline{q}, \underline{\theta}\right) \\ &= C\left(\underline{q}, \underline{\theta}\right) + C\left(\overline{q}, \overline{\theta}\right) - C\left(\overline{q}, \underline{\theta}\right).\end{aligned}$$

- Plugging these values of $\overline{t}$ and $\underline{t}$ into P's objective function:

$$\begin{aligned} V &= \nu\left[S\left(\underline{q}\right)-\underline{t}\right]+(1-\nu)\left[S\left(\overline{q}\right)-\overline{t}\right] \\ &= \nu\left[S\left(\underline{q}\right)-C\left(\underline{q},\underline{\theta}\right)-C\left(\overline{q},\overline{\theta}\right)+C\left(\overline{q},\underline{\theta}\right)\right]+(1-\nu)\left[S\left(\overline{q}\right)-C\left(\overline{q},\overline{\theta}\right)\right] \end{aligned}$$

- P's problem is now to maximize the objective V above with respect to only two choice variables, $\underline{q}$ and $\overline{q}$.

- The first-order condition with respect to $\underline{q}$:

$$\frac{\partial V}{\partial \underline{q}}=\nu\left[S'\left(\underline{q}\right)-C_q\left(\underline{q},\underline{\theta}\right)\right]=0 \Rightarrow S'\left(\underline{q}^{SB}\right)=C_q\left(\underline{q}^{SB},\underline{\theta}\right).$$

  - This means that $\underline{q}^{SB}=\underline{q}^{FB}$, as we were asked to show.

- The first-order condition with respect to $\overline{q}$:

$$\frac{\partial V}{\partial \overline{q}}=\nu\left[-C_q\left(\overline{q},\overline{\theta}\right)+C_q\left(\overline{q},\underline{\theta}\right)\right]+(1-\nu)\left[S'\left(\overline{q}\right)-C_q\left(\overline{q},\overline{\theta}\right)\right]=0$$

or

$$(1-\nu)\,S'\left(\overline{q}\right)=(1-\nu)\,C_q\left(\overline{q},\overline{\theta}\right)+\nu\left[C_q\left(\overline{q},\overline{\theta}\right)-C_q\left(\overline{q},\underline{\theta}\right)\right]$$

or

$$S'\left(\overline{q}^{SB}\right)=C_q\left(\overline{q}^{SB},\underline{\theta}\right)+\frac{\nu}{1-\nu}\left[C_q\left(\overline{q}^{SB},\overline{\theta}\right)-C_q\left(\overline{q}^{SB},\underline{\theta}\right)\right].$$

  - From the last equality we see that $\overline{q}^{SB}<\overline{q}^{FB}$ if and only if the last term on the right-hand side is strictly positive. We can write:

$$\frac{\nu}{1-\nu}\left[C_q\left(\overline{q}^{SB},\overline{\theta}\right)-C_q\left(\overline{q}^{SB},\underline{\theta}\right)\right]>0 \Leftrightarrow C_q\left(\overline{q}^{SB},\overline{\theta}\right)-C_q\left(\overline{q}^{SB},\underline{\theta}\right)>0 \Leftrightarrow$$

$$\int_{\underline{\theta}}^{\overline{\theta}} C_{q\theta}\left(\overline{q}^{SB},\theta\right)d\theta>0,$$

    which always holds due to the assumptions that $\overline{\theta}>\underline{\theta}$ and $C_{q\theta}>0$. This means that we indeed have $\overline{q}^{SB}<\overline{q}^{FB}$, as we were asked to show.

d) Explain the intuition for the results you were asked to show under c). Also explain the nature of the trade-off that the principal faces.

- The trade-off that the principal faces when solving the problem under asymmetric information is between, on the one hand, letting the agent types produce the efficient levels and, on the other hand, not to give away rents to the agent.
- The reason why P cannot achieve both those goals is that he cannot observe A's type. In particular, if P offered contracts that involved full efficiency and no rent extraction, then the good type of agent would have an incentive to choose the contract aimed at the bad type of agent (so IC-good would be violated).

- In order to make sure that IC-good is satisfied, P can do two things.
  - First, he can make the bad type's contract less attractive in the eyes of the good type by asking the bad type to produce less (so a quantity below the efficient level). If doing that, P would need to pay less money to the bad type (to ensure that his IR constraint is satisfied), which makes the bad type's contract less attractive.
  - Second, P can make the payment in the good type's contract larger, which again would lower the good type's incentive to choose the bad type's contract.
  - P will find it optimal to do a little bit of both those things, thus distorting the bad type's quantity downwards and giving away some rents to the good type.

From the lecture slides:

- Key to the results we have derived is that the good type is the one who gets, for any given q, both:

  (i) the highest *marginal* utility [due to Spence-Mirrlees] and

  (ii) the highest *total* utility.

- Because of (ii), the principal's top priority is to make the good type choose his first-best quantity.
  - That type can get a high utility level (relative to the outside option utility), which the principal then can grab a large part of.

- If the principal were to take too much of the good type's utility, then that type would instead choose the bad type's bundle.
  - To prevent this, the principal makes the bad type's bundle less attractive by lowering that type's quantity and payment.
  - This way of separating the two types works because of (i): *The good type benefits less from a reduction in q than the bad type.*

Question 2 (moral hazard)

This is a model of so-called sharecropping that lets the farmer's effort choice be continuous. It builds on similar model from the course, which assumed a binary effort choice.

A landlord (the principal, P) owns a piece of land and wants to lease the land to a poor farmer (the agent, A). If entering such an agreement, A will, when farming the land, choose what effort to make, $e \in [0,1]$. The associated effort cost equals $\psi(e)$, where this function satisfies

$$\psi' > 0, \quad \psi'' > 0, \quad \psi(0) = \psi'(0) = 0, \quad \lim_{e \to 1} \psi'(e) = \infty.$$

Depending on A's effort and on the weather, the output that is produced may be high ($q = \overline{q}$) or low ($q = \underline{q}$, with $0 \leq \underline{q} < \overline{q}$). The probability that output is high equals the effort level: $\Pr(q = \overline{q} \mid e) = e$. The market price of the output equals unity. Therefore, q is also the market value of the output.

P (and the court) can observe which quantity that is realized ($\overline{q}$ or $\underline{q}$) but not whether A has worked hard or not. Therefore, in principle, the contract between P and A could consist of two numbers, indicating how much A should pay P in each state (a high-output state or a low-output state). However, the contract that is actually used is a so-called sharecropping contract, which is characterized by a single number, $\alpha \in [0,1]$. The number α is the share of output that A is allowed to keep, whereas the remaining share $1-\alpha$ is paid to P. Therefore, P's expected profit equals

$$V = (1-\alpha)\left[e\overline{q} + (1-e)\underline{q}\right].$$

Moreover, A's expected utility equals $U = \alpha\left[e\overline{q} + (1-e)\underline{q}\right] - \psi(e)$. A's outside option would yield the utility zero. A is protected by limited liability, meaning that a contract cannot stipulate that A must pay, in net terms, some amount of money to P. It is assumed that P has all the bargaining power and makes a take-it-or-leave-it offer to A.

a) **Characterize the second best optimal values of α and e, using the first-order approach. Assume that the functional forms are such that the second-order conditions are satisfied.**

- It suffices to characterize the optimal second best values of e and α assuming that P indeed wants to implement a strictly positive effort level.
  - However, any attempts to show that setting $e = 0$ is suboptimal, or a discussion of this issue, are given credit.
- P's problem can be written as: Maximize

$$V = (1-\alpha)\left[e\overline{q} + (1-e)\underline{q}\right]$$

w.r.t. e and α, subject to A's incentive compatibility (IC) constraint

$$e \in \arg\max_{e' \in [0,1]} \left\{ \alpha \left[e' \overline{q} + (1 - e') \underline{q} \right] - \psi(e') \right\} \tag{1}$$

and A's individual rationality (IR) constraint

$$\alpha \left[e \overline{q} + (1 - e) \underline{q} \right] - \psi(e) \geq 0.$$

The limited liability (LL) constraint is automatically satisfied given that A's share of production is assumed to be non-negative ($\alpha \geq 0$).

- We can note that A's payoff U evaluated at $e = 0$ is non-negative:

  $$U \mid_{e=0} = \alpha \underline{q} - \psi(0) = \alpha \underline{q} \geq 0.$$

  Therefore, if e (for any value of α) is chosen optimally by A, which is exactly what IC requires, then U cannot be negative and the IR constraint is hence automatically satisfied.

  – P's problem thus amounts to maximizing V w.r.t. e and α, subject only to A's IC constraint.

- The idea behind the first-order approach is to replace the infinitely many IC constraints in (1) with the single constraint that the first-order condition associated with A's problem of choosing e must be satisfied. We can write A's problem as

  $$\max_{e \in [0,1]} \alpha \left[e \overline{q} + (1 - e) \underline{q} \right] - \psi(e).$$

  The first-order condition:

  $$\alpha \left(\overline{q} - \underline{q} \right) = \psi'(e). \tag{*}$$

  (We know that if $\alpha > 0$, then the e defined by (*) satisfies $e \in (0,1)$ because of the assumptions that $\psi'(0) = 0$ and $\lim_{e \to 1} \psi'(e) = \infty$. If P wants to implement a strictly positive effort level, then he must choose some $\alpha > 0$.)

- P's problem can now be written as: Maximize

  $$V = (1 - \alpha) \left[e \overline{q} + (1 - e) \underline{q} \right]$$

  w.r.t. e and α, subject to A's IC constraint:

  $$\alpha \left(\overline{q} - \underline{q} \right) = \psi'(e). \tag{IC}$$

  Plug the IC into the objective function:

  $$\begin{aligned} V &= (1 - \alpha) \left[e \overline{q} + (1 - e) \underline{q} \right] \\ &= \left[1 - \frac{\psi'(e)}{\overline{q} - \underline{q}} \right] \left[e \overline{q} + (1 - e) \underline{q} \right]. \end{aligned}$$

- Take the first-order condition w.r.t. e:

$$\frac{\partial V}{\partial e} = -\frac{\psi''(e)}{\overline{q}-\underline{q}}\left[e\overline{q}+(1-e)\underline{q}\right] + \left[1-\frac{\psi'(e)}{\overline{q}-\underline{q}}\right]\left(\overline{q}-\underline{q}\right) = 0$$

or

$$\frac{\psi''(e)}{\overline{q}-\underline{q}}\left[e\overline{q}+(1-e)\underline{q}\right] = \left[1-\frac{\psi'(e)}{\overline{q}-\underline{q}}\right]\left(\overline{q}-\underline{q}\right)$$

or

$$\psi''\left(e^{SB}\right)\left[e^{SB}\overline{q}+\left(1-e^{SB}\right)\underline{q}\right] = \left[1-\frac{\psi'\left(e^{SB}\right)}{\overline{q}-\underline{q}}\right]\left(\overline{q}-\underline{q}\right)^2.$$

  - The second-order condition:

$$\frac{\partial^2 V}{\partial e^2} = -\frac{\psi'''(e)}{\overline{q}-\underline{q}}\left[e\overline{q}+(1-e)\underline{q}\right] - \frac{2\psi''(e)}{\overline{q}-\underline{q}}\left(\overline{q}-\underline{q}\right) < 0$$

or

$$-\frac{\psi'''(e)\left[e\overline{q}+(1-e)\underline{q}\right]}{\overline{q}-\underline{q}} - 2\psi''(e) < 0.$$

In the question it is said that we are supposed to assume that the functional forms are such that this condition is satisfied (assuming, on top of the previous assumptions, that $\psi''' \geq 0$ would be sufficient, but not necessary).

- We can obtain the second-best optimal level of α by evaluating (*) at $e = e^{SB}$:

$$\alpha^{SB} = \frac{\psi'\left(e^{SB}\right)}{\overline{q}-\underline{q}}.$$

- Summing up: the second-best levels of e and α are implicitly defined by

$$\psi''\left(e^{SB}\right)\left[e^{SB}\overline{q}+\left(1-e^{SB}\right)\underline{q}\right] = \left[1-\frac{\psi'\left(e^{SB}\right)}{\overline{q}-\underline{q}}\right]\left(\overline{q}-\underline{q}\right)^2$$

and

$$\alpha^{SB} = \frac{\psi'\left(e^{SB}\right)}{\overline{q}-\underline{q}}.$$

b) In a richer model with both a continuum of effort levels and output levels, what are the two conditions needed to ensure that the first-order approach is valid? You do not have to state the conditions formally — it suffices if you do it in words.

- The two (jointly) sufficient conditions are:
  - The probability distribution that describes the mapping from effort to outcome satisfies the monotone likelihood rate property (MLRP):

$$\frac{\partial}{\partial q}\left(\frac{f_e(y \mid e)}{f(y \mid e)}\right) > 0.$$

– The CDF of that function is convex in effort:

$$F_{ee}(y \mid e) > 0.$$

c) **Instead of the model discussed above, consider the following moral hazard model with a risk neutral principal and a risk neutral agent who is protected by limited liability. There are two effort levels (0 and 1) and four output levels (y_1, y_2, y_3 and y_4). The probabilities with which the different output levels realize, given the two different effort levels, are indicated in the following table:**

	Effort = 0	Effort = 1
y_1	$\pi_{10} = 0.3$	$\pi_{11} = 0.1$
y_2	$\pi_{20} = 0.4$	$\pi_{21} = 0.4 + 2x$
y_3	$\pi_{30} = 0.2$	$\pi_{31} = 0.3 - x$
y_4	$\pi_{40} = 0.1$	$\pi_{41} = 0.2 - x,$

where $x \in [0, 0.1]$. What is the condition that we need to impose on the model to ensure that the principal's optimal contract is such that the agent's payment is strictly increasing in the level of output that is realized? For what values of x is this condition satisfied? Explain the intuition for why this condition matters.

- The condition is called the monotone likelihood ratio property (MLRP). It requires that the so-called likelihood ratio — defined as $\frac{\pi_{i1}}{\pi_{i0}}$ — is increasing in the output level.
  - [The book uses a slightly different definition ($\frac{\pi_{i1}-\pi_{i0}}{\pi_{i1}}$), and that would also be a valid answer.]
- For MLRP to be satisfied in this example the following three inequalities must be satisfied:

  $$\frac{\pi_{21}}{\pi_{20}} > \frac{\pi_{11}}{\pi_{10}} \quad \text{and} \quad \frac{\pi_{31}}{\pi_{30}} > \frac{\pi_{21}}{\pi_{20}} \quad \text{and} \quad \frac{\pi_{41}}{\pi_{40}} > \frac{\pi_{31}}{\pi_{30}}.$$

  The first inequality is obviously satisfied for all positive values of x. The second inequality is satisfied if

  $$\frac{\pi_{31}}{\pi_{30}} > \frac{\pi_{21}}{\pi_{20}} \Leftrightarrow \frac{0.3 - x}{0.2} > \frac{0.4 + 2x}{0.4} \Leftrightarrow 2(0.3 - x) > 0.4 + 2x \Leftrightarrow x < \frac{0.2}{4} = 0.05.$$

  The third inequality is satisfied if

  $$\frac{\pi_{41}}{\pi_{40}} > \frac{\pi_{31}}{\pi_{30}} \Leftrightarrow \frac{0.2 - x}{0.1} > \frac{0.3 - x}{0.2} \Leftrightarrow 2(0.2 - x) > 0.3 - x \Leftrightarrow x < 0.1.$$

  Summing up: the question constrains x to be in the interval $[0, 0.1]$; in addition we must impose the constraint $x < 0.05$ for MLRP to be satisfied.
  - [In the course we haven't been very careful about if/when we should have weak or strict inequalities, and this issue is not important for a valid answer. However, the student should be consistent within her or his own answer.]

- The intuition for why MLRP matters can be understood as follows. When choosing which output level to reward with a relatively large payment, the principal will consider the effect this choice has on the agent's incentive to choose the high effort level. In order to make that incentive as strong as possible, the principal should reward an output level for which choosing a high effort has a big (positive) impact on the probability of obtaining that output level. To see this, suppose $x = 0.1$ in the above example. Then the highest output level (y_4) would realize with the same probability (namely, 0.1), regardless of whether the agent exerts a low or a high effort. Therefore, rewarding the highest output level with a large payment would certainly not create an incentive for the agent to choose the high effort. However, if the principal awarded the second output level (y_2) with a large payment, then the agent would know he could increase the likelihood of receiving that payment with a positive amount (from 0.4 to 0.6) by choosing to make a high effort, and his incentive to do so would therefore be strong. In other words, the principal will have an incentive to reward an outcome for which π_{i1} (the high-effort probability) is large relative to π_{i0} (the low-effort probability) — formal analysis shows that it's the ratio that matters. As the MLRP condition guarantees that this ratio in larger for high output levels than for lower ones, it ensures that the payments also are higher (or at least not lower) for higher output levels.
  - Yet another way of phrasing an intuitive explanation is to say that the principal should reward an outcome whose realization is *informative* about what effort the agent has chosen. That is, the principal should reason like an econometrician who calculates a maximum likelihood estimator — he should reward the outcome that maximizes the likelihood, conditional on the principal's observing that output, that the agent has exerted a high effort.

END OF EXAM